

Polynomials

Factorising Polynomials

2003 P2	<p>1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.</p> <p>(a) Show that $(x - 2)$ is a factor of $f(x)$.</p> <p>(b) Express $f(x)$ in its fully factorised form.</p>	4
2015 SP P1	<p>7. (a) Show that $(x + 1)$ is a factor of $x^3 - 13x - 12$.</p> <p>(b) Factorise $x^3 - 13x - 12$ fully.</p>	3 2
2015 P1	<p>3. Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully.</p>	4

Pre 2000 – Factorising Polynomials

1	Factorise fully $2x^3 + 5x^2 - 4x - 3$.	4
2	<p>(a) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$.</p> <p>(b) Hence find the other roots.</p>	1 3
3	Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form.	4
4	Express $x^4 - x$ in its fully factorised form.	4
5	<p>(a) Show that $(x - 3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$.</p> <p>(b) Hence express $f(x)$ in its fully factorised form.</p>	2 2
6	<p>(a) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$.</p> <p>(b) Show algebraically that there are no other real roots.</p>	2 3
7	<p>(a) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$. The function g is defined by $g(x) = x - 1$. Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$.</p> <p>(b) Factorise fully $f(g(x))$.</p> <p>(c) The function k is such that $k(x) = \frac{1}{f(g(x))}$. For what values of x is the function k not defined?</p>	(4) (3) (2)

Finding Coefficients

3. (JAN) 02 P1	Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d .	5
2014 P1	<p>22. For the polynomial $6x^3 + 7x^2 + ax + b$,</p> <ul style="list-style-type: none"> $x + 1$ is a factor 72 is the remainder when it is divided by $x - 2$. <p>(a) Determine the values of a and b.</p> <p>(b) Hence factorise the polynomial completely.</p>	4 3

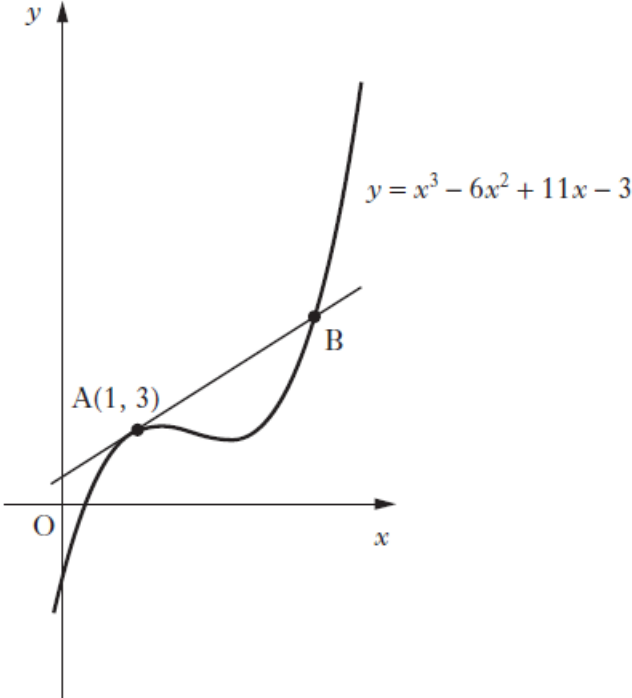
Pre 2000 – Finding Coefficients

1	One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3 . Find the value of p and the other roots.	4
2	Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$.	3
3	When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x - 2)$, the remainder is 114. One factor of $f(x)$ is $(x - 2)$. Find the values of p and q .	5
4	Find p if $(x + 3)$ is a factor of $x^3 - x^2 + px + 15$	3

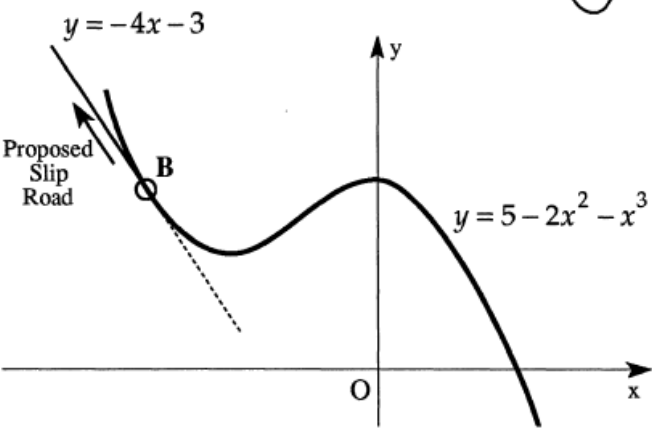
Solving Polynomial Equations

2001 P2	<p>1. (a) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k.</p> <p>(b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value.</p>	3 2
2004 P1	<p>2. $f(x) = x^3 - x^2 - 5x - 3$.</p> <p>(a) (i) Show that $(x + 1)$ is a factor of $f(x)$. (ii) Hence or otherwise factorise $f(x)$ fully.</p> <p>(b) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point.</p>	5 1
2005 P2	<p>11. (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$.</p> <p>(b) Hence find the range of values of p for which all the roots of the cubic equation are real.</p>	1 7

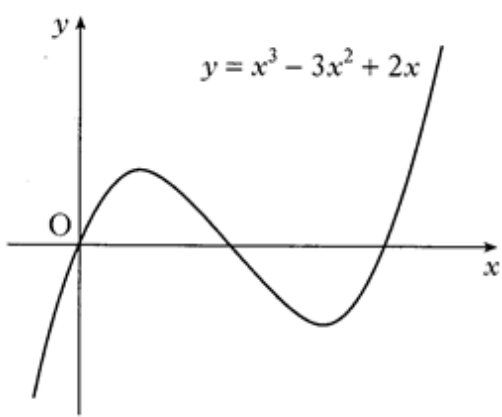
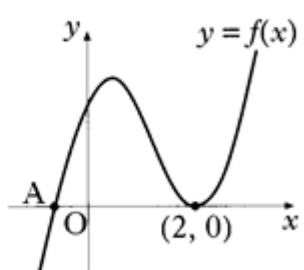
2008 SP P1	<p>23. The diagram shows a sketch of functions f and g where $f(x) = x^3 + 5x^2 - 36x + 32$ and $g(x) = -x^2 + x + 2$.</p> <p>The two graphs intersect at the points A, B and C.</p> <p>Determine the x-coordinate of each of these three points.</p> <div data-bbox="917 114 1295 584"> </div> <div data-bbox="1385 360 1401 387" style="text-align: right;">8</div>
2010 P1	<p>22. (a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.</p> <p>(ii) Hence factorise $f(x)$ fully.</p> <p>(b) Solve $2x^3 + x^2 - 8x + 5 = 0$.</p> <p>(c) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.</p> <p>Find the coordinates of G.</p> <p>(d) This tangent meets the curve again at the point H.</p> <p>Write down the coordinates of H.</p> <div data-bbox="1425 674 1441 701" style="text-align: right;">5</div> <div data-bbox="1425 745 1441 772" style="text-align: right;">1</div> <div data-bbox="1425 902 1441 929" style="text-align: right;">5</div> <div data-bbox="1425 1025 1441 1052" style="text-align: right;">1</div>
2011 P2	<p>2. Functions f, g and h are defined on the set of real numbers by</p> <ul style="list-style-type: none"> • $f(x) = x^3 - 1$ • $g(x) = 3x + 1$ • $h(x) = 4x - 5$. <p>(a) Find $g(f(x))$.</p> <p>(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$.</p> <p>(c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.</p> <p>(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.</p> <p>(d) Hence solve $g(f(x)) + xh(x) = 0$.</p> <div data-bbox="1425 1312 1441 1339" style="text-align: right;">2</div> <div data-bbox="1425 1384 1441 1411" style="text-align: right;">1</div> <div data-bbox="1425 1507 1441 1534" style="text-align: right;">5</div> <div data-bbox="1425 1579 1441 1606" style="text-align: right;">1</div>

2015 OLD P1	<p>21. (a) Show that $(x - 1)$ is a factor of $x^3 - 6x^2 + 9x - 4$ and hence factorise $x^3 - 6x^2 + 9x - 4$ fully. 4</p> <p>(b) The diagram shows the graph with equation $y = x^3 - 6x^2 + 11x - 3$.</p>  <p>(i) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 11x - 3$ at the point A(1, 3). 3</p> <p>(ii) Hence find the coordinates of B, the point of intersection of this tangent with the curve. 3</p>
2015 SP P1	<p>2. Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$. 5</p>
2015 EP P1	<p>5. For the polynomial, $x^3 - 4x^2 + ax + b$</p> <ul style="list-style-type: none"> • $x - 1$ is a factor • -12 is the remainder when it is divided by $x - 2$ <p>(a) Determine the values of a and b. 4</p> <p>(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. 4</p>
2017 P2	<p>2. (a) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 - 5x^2 + x + 2$. 2</p> <p>(b) Hence, or otherwise, solve $f(x) = 0$. 3</p>

Pre 2000 Questions – Solving Polynomial Equations

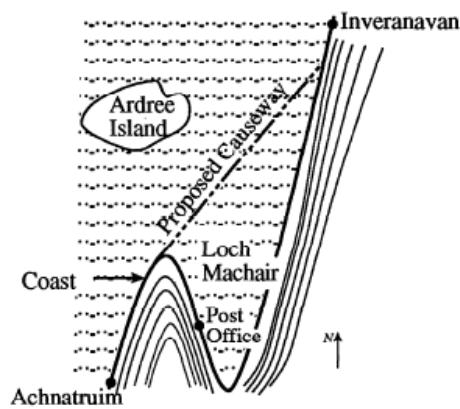
1	<p>The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).</p> <p>(a) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. (3)</p> <p>(b) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. (4)</p>
2	<p>The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.</p> <p>Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.</p>  <p>(a) Calculate the coordinates of B. (7)</p> <p>(b) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. (1)</p>
3	<p>(a) (i) Make a sketch of the graph of $y = x^3$, where $-3 \leq x \leq 3$, $x \in \mathbf{R}$. (3)</p> <p>(ii) On the same diagram, draw the graph of $y = 6x + 1$. (3)</p> <p>(b) State the number of roots which the equation $x^3 = 6x + 1$ has in the interval $-3 \leq x \leq 3$. (1)</p> <p>(c) Calculate the value of the positive root, correct to 3 significant figures. (4)</p>

Polynomials and Differentiation

2000 P2	<p>A1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.</p> <p>(a) Find the equation of the tangent to this curve at the point where $x = 1$.</p> <p>(b) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.</p>		5 5
2002 P2	<p>3. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.</p> <p>(a) Find the x-coordinate of the maximum turning point.</p> <p>(b) Factorise $2x^3 - 7x^2 + 4x + 4$.</p> <p>(c) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$.</p>		5 3 2
2005 P1	<p>8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.</p> <p>(a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.</p> <p>(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.</p> <p>(c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.</p>	5 2 5	
2008 P1	<p>21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.</p> <p>(a) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature.</p> <p>(b) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully.</p> <p>(c) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.</p>	6 5 4	
2013 P2	<p>3. (a) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.</p> <p>(b) Show that the curve with equation</p> $y = x^4 + 4x^3 + 2x^2 - 20x + 3$ <p>has only one stationary point.</p> <p>Find the x-coordinate and determine the nature of this point.</p>	4 5	

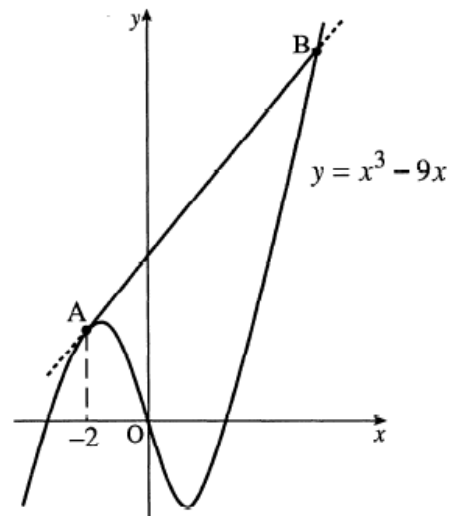
1

The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB.

The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



- (a) (i) Write down the coordinates of A.
(ii) Find the equation of the line AB.

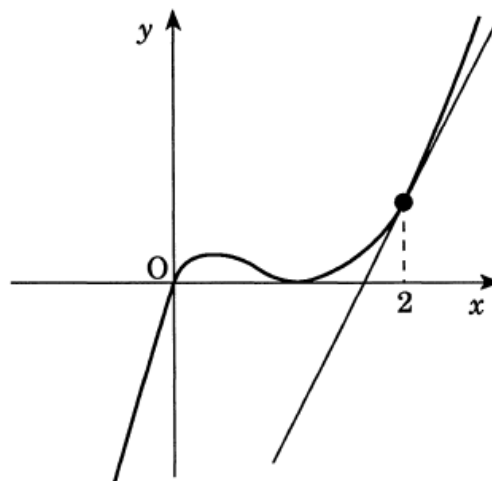
(5)

- (b) Determine the coordinates of the point B which represents the northern end of the causeway.

(7)

2

The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.

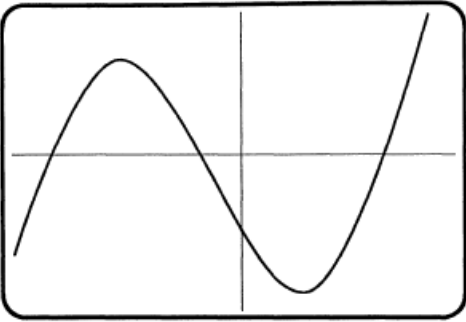


- (a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$.

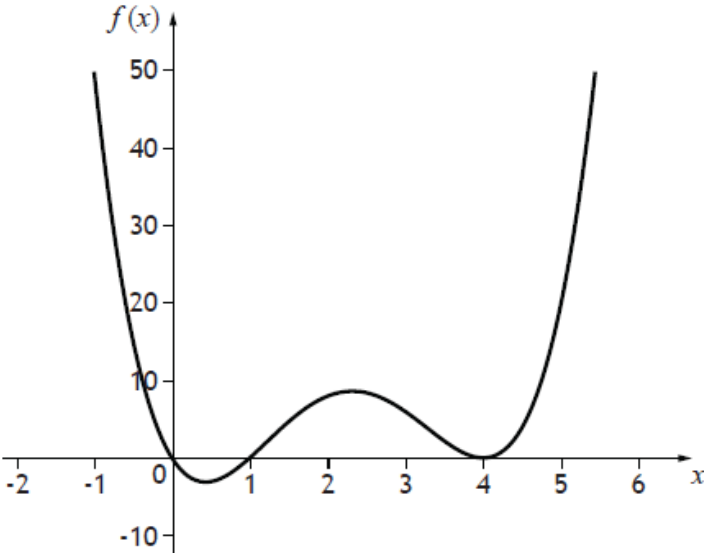
(4)

- (b) Find algebraically the coordinates of the point where this tangent meets the curve again.

(5)

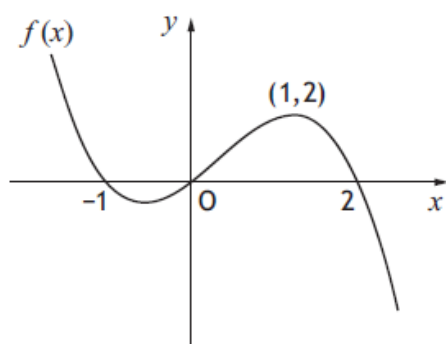
3	<p>The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.</p>  <p>(a) Factorise $f(x)$. (3)</p> <p>(b) Write down the co-ordinates of the four points where the curve crosses the x and y axes. (2)</p> <p>(c) Find the turning points and justify their nature. (6)</p>
---	---

Polynomial Functions from Graphs

2015 SP P2	<p>3. The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.</p>  <p>(a) Determine the values of p and q. 1</p> <p>(b) Find the equation of the tangent to the curve when $x = 1$. 4</p>
------------------	---

2015
EP
P1

2. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.
The curve passes through $(-1,0)$, $(0,0)$, $(1,2)$ and $(2,0)$.

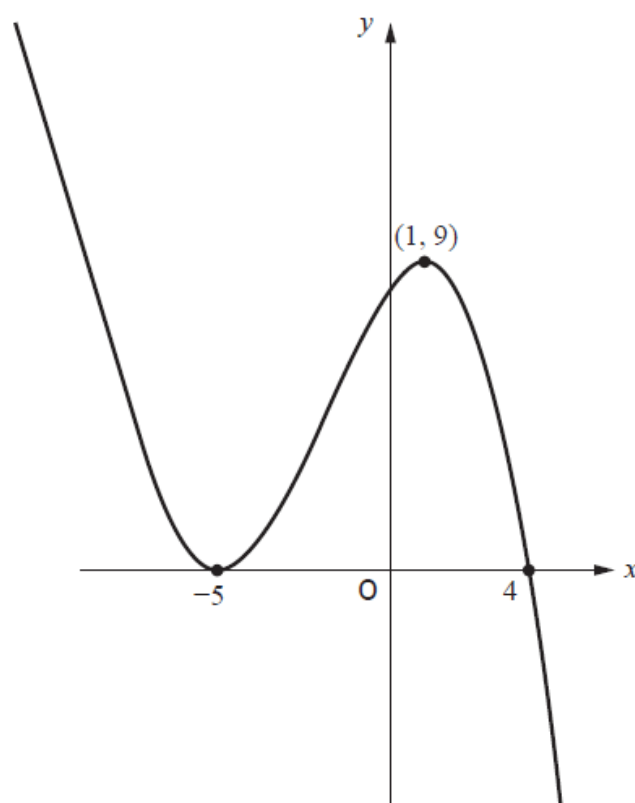


Find the values of a , b and k .

3

2016
P1

15. The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.



- (a) Find the values of a , b and k .

3

- (b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.

1

Approximate Roots

4.
(JAN)
02 P2

The graph of $f(x) = 2x^3 - 5x^2 - 3x + 1$ has been sketched in the diagram shown.

Find the value of a correct to one decimal place.

